The Analysis of Hidden Geometric of Persian Painting, 
Case Study: One Piece Miniature of Kamal Aldin Bihzad

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Abstract

Hidden geometry is a set of geometric coordinates based on which the artist composes the artwork, and its object is to compile a homogenous unit that conveys artistic expression. This geometry and its rational scales is named “hidden geometry” after its invisibility in the work and also named “fundamental geometry” because the work is founded in accordance with that. In studying the hidden geometry, in fact, the geometric principles are investigated based on which the artist composed the artwork. This study investigates the hidden geometry of an artworks, a miniature of Timurid Zafarnameh which is preserved in Gulistan Palace Museum (a celebration after Timur’s return to Samarqand) and composed by an Iranian Miniaturist, Master Kamal al-Din Bihzad, by considering the accurate principles of scales, geometric schema, forms and harmony in the work segments, and tries to answer the following questions: how is the hidden geometry procedure of composing in miniature of Bihzad, and if it is possible to characterize the scales and harmony between the work’s geometric infrastructure and theme by using geometric analysis. Thus it is important to characterize the geometric infrastructure of Bihzad’s work which this paper tries to study. On the other hand studying the hidden geometry in works of Iran’s miniature masters is helpful for today’s miniature creating. The works' formative analysis is an operation that focuses on ostensible structure and its specifications. Here the geometric analysis of figure or form will be considered as functional investigation of conventions that merely depict the ostensible aspects of a work; hence the geometric fundamental theories will be taken in consideration for characterizing the scales and hidden geometry of Bihzad’s work. Consequently, by using the geometric coordination and rational scales, it is revealed that, for creating supreme artworks, Bihzad has used some scales based on irrational-numbers and Golden ratio (φ), in addition to circle and quadrangle based spirals; it demonstrate evolutionary procedure of composition in Persian Painting and reaching the soar of compilation in miniature infrastructure through him.

Keywords: Persian Painting, Kamal al-Din Bihzad, Composition, Hidden Geometry

1. The current work has been orally presented in International Conference on Orientalism, Persian History and Literature and anniversary of SaadiShirazi (26th May 2016)
It reveals its truth via some components without shattering its unity.
Frithjof Schuon

1. Research history

When searching in the libraries, we encounter the name "Kamal al-din Bihzad" frequently. There are influential books and articles which present the biography of Kamal al-din Bihzad, introduce his works that follow Herat and Tabriz schools, provide the reflection of his views and beliefs in his works, investigate the realistic and naturalistic aspects of his works, offer aesthetic interpretations of his works based on theosophical concepts, etc. The theoretical aspect of the hidden geometry has been studied in some ways. But, the application of the hidden geometry in Bihzad's paintings was ignored and not analyzed carefully. Among these books and articles, there are a few which have worked on such a theme; yet, these did not devote much space to geometric aspect. Firstly, we can mention the article "Hidden Geometry in the Paintings of Kamal al-din Bihzad" by Khashayar Ghazi Zadeh (2003), printed in "Khial Quarterly"; the article, despite its name's generality, investigates only two works by Bihzad, "Yusef and Zuleykha" and "Whirling dervishes". The other one is an article from the magazine "the month's book of art" with the title "Demythologization and Myth-making of Castle Khovarnaq from the viewpoint of Nizami and Bihzad" by Bahman Namvar Motlagh (2003) whose title is essentially different. That’s why the composition of the mentioned work is discussed to some extent in one of the sections of the article. However, the present study aims to analyze one of the paintings of Master Kamal al-din Bihzad selected from Timurid Zafarnameh. Those studies are different from this one in that they either discuss the composition of the works briefly or consider the example concepts of the paintings or its relationship with the theosophical and personal opinions; the works' compositions were not analyzed geometrically. Finally, the results were not scientific, mathematical (geometric), and attributable. Studying the way painters, depicting Persian Painting atmosphere, have reached a composition of visual elements based on mathematical logic is the theme developed based on researches on fundamental and theoretical concepts of the geometric spaces/shapes in mathematics and its relationship with visual arts and rules. Regarding the limitations and the lack of frequent and comprehensive references in the field (while Western Paintings have been analyzed in detail), it will be of use as a guideline for understanding the importance of the geometric dimensions of the composition of Persian Paintings.

2. Methodology

Such researches conducted in order to understand the phenomena and in other words they aim to test theories, explain the relationship between phenomena, conduct follow-up studies based on the present researches, make the relations in the infrastructures clear, and finally, continue and complete the previous researches. The method is qualitative descriptive-analytic one (geometric identification and analysis). Data was collected by referring to specialized library resources (attributable), visual archives and articles of museums, and valid scientific databases.

What identify the artwork in many formalistic researches are forms and structures; such studies only investigate these. Concerning visual arts, formal values not only include balance, symmetry, and perspective but they are also the result of the arrangement of shapes and lines. Sometimes, a combination and order of colors makes a work balanced and symmetric. Despite the diversity of those which are described as formal, all the examples have one thing in common; the relationship between specific components or certain characteristics (Shepard, 2011). Soon after formalism, another point of view emerged in the field of scientific studies called new formalism. According to this theory, the most significant drawback of the formalistic views on art is its dramatic ignorance of the link between the content and the dignity of art. Hence, to amend it, an obvious way is to find an agreement between these two. New formalists consider the content of a work as the needed condition for art. They suggest that in an artwork, if it is really a work of art, the form and content are surely and appropriately interrelated (Carroll, 2011) Today, art theorists and historians believe that form and content are not separable (Liedman, 2000). And what expresses or indicates painting and music is the content not the form. In this way, this paper examines the question of coordination and the link between content and form by applying a new formalistic approach.
2. Conclusion

Admitting that Persian Painting follows some hidden rules in its structure cannot be a good criterion to differentiate rule-governed works from impromptu paintings. However, it is the geometric analysis of samples which enables us to think useful thoughts about proving its existence. In the structure of a great painting all the elements of the image, in connection with one another, present an appropriate sample. In fact, the unifying role of the form develops a kind of order in all of its aspects which despite the multiplicity and diversity of the elements, such as the harmony of sounds in a piece of music which has been set for a concert, directs all the components in parallel and under a unified form.

An immediate thought enables us to find out that, as the examined images have visible and hidden structures, exact coordination of the existing elements is not possible when there is no specific rule for developing the composition. The hidden structure, which is the essence of the inside coordination, is known only to the artist; it is not expressed in the image but kept in accordance with the lines and shapes and dominant on their relations in each phase of the creation of the painting to be implied in its final order and glory. Mentioning some examples of scales and logical relations may help to express that shaping what has happened in geometric relations is rule-governed. In figure 7, the guidelines of each of the rectangles is beautifully in accordance with the beginning and the end of the inscription, the door and the widows, and king's position; they are also the determinants of the place of sketching each of the figures. In figure 8, we can refer to the equality of the measures of the width of the standard image (the king's) with the width of the windows, the distance between each pair of other figures, the width and the length of the visible part of the roof and also equality of length of the standard image with the length of the windows or the length of standing figures. In figure 10, the width of the first horizontal rectangle's borders (a piece of writing) on the top of the composition is determined by the distance between the beginning of the Ghezelbash hat's rod to the end of the feather and the width of the second horizontal rectangle's borders (another piece of writing) on the top of the composition is equal to the distance between the end of the hat's feather to the beginning of the face. Even the position of the figures of the king, two singers and the group of the courtiers have been arranged purposefully and parallel to seated direction of the standard figure. In figure 11, too, the coordination of distances between the starting point of the forehead to where the legs are bent in the sitting position (land line) with the length of the main and central border of the composition is obvious. These proportions exist in other borders, too.

Finally, we can examine the results obtained from investigating this painting's ostensible identity in two groups or approaches. Making use of the theories and achievement of both of them is essential for adopting a comprehensive view about the state of geometric system in Persian Paintings. In the first group, (The answer to the first question which was raised at the beginning of this research) the relations among the visual elements of Persian Painting are not shaped only to represent the metaphysical world and stuff like that. Rather, it is also in connection with geometric and mathematical rules in order to indicate the importance of sciences and the artist's awareness of the knowledge of geometry and its determinant role in designing them. In the second group, (the answer to the second question) which is interpreted based on identifying the situation and the political, social, historical and cultural environment of the juncture of the work's formation, the richness of a character is the cause for his being affective in the society from different aspects such as the field of art; in a way that this inspiration reveals not only in choosing the paintings' topics which are mainly about the king and his life events but also in the hidden structure of the works. It shows that the royal and luxurious aspect of Persian Painting is bolder than its theosophical aspect. The trick which was employed here, i.e. using the scales and measures of an element as the criterion to determine the other minor and major measures in the composition is a clever idea. The artist follows his understanding of creation and the current rules in the universe by shaping his own rule-governed infrastructure with the help of the measures of the images and indirectly represents his concepts and beliefs by processing his ideas with the help of images. The painter develops the needed scales among the measures of the image of the king when sketching it in one phase and in the next phase, at the macro level, he develops this order and unity all over the composition. As a result, the whole painting is presented containing a series of coordinated measures which are dependent on each other in order to create a work in coordination with the unifying rule of the universe by extracting a visible pattern resulted from a hidden infrastructure.
Finally, relying on what was mentioned, it can be concluded that the used principles in the geometric structure of a work is completely in accordance with and dependent on its theme to the extent that the existing scales of the main character was used as a criterion and a basis for sketching the measures and the distances in the whole work since painters were able to use all the available tools. Among these, the most important one was the image of the king itself in order to determine the scales among minor and major measures of the figure smartly, in the micro level and at the macro level, to extend these proportional measures to other existing measures in the composition (the representation of the idea of oneness in multiplicity and multiplicity in oneness visually and geometrically expressed) because the king is not only the main hero of the miniature but also the most important and effective political and social character of his era. The king, whose soul attracted divine light and was full of divine grace, is the most excellent embodiment of absolute harmony and after that the theory of "the king as the world's Qibla" or the king as the center of the world and the one who develops the order and the connections among the affairs of the universe. Therefore, the idea of the harmony of the creation should be expressed quite clearly in the image of the king so that the king will be introduced as the noblest creature and the source of the truth of the world. As we know, in ancient cultural models in Mesopotamia, too, the king had the highest rank and also, in other ancient cultures, people considered a divine position for him; and the one who was on the throne was seen as an embodiment of God (the God's representative). This holy face can only appear in the center of the universe and the whole world would expand around him. Therefore, here we are facing a system that is in harmony with the system of the cosmos. As different religions believe it, the most sacred spot in the circle of existence is its center (according to the Islam Kaabah is the core and the most sacred place on the earth with holy geometric dimensions since Polaris is an evidence which justifies that Kaabah is in front of the center of the sky). This concept is the same symbolism of the center and justifies the belief that the main spot, with its hidden geometric principles and relations in the center, makes the unbroken and accurate connections among other points. The same concept has been repeated in the world of Persian Painting; as the universe expands from a central spot in all directions, the center of the painting is specific to the main figure, i.e. the king, that arrange the other geometric scales with its own measures. Thus, the scales among the measures come from anthropometric (the system that is dependent on the size of the human body) into kingmetric and comes to be dependent on the existing proportions among measures of the king himself which itself is a type of anthropocentric (believing in human as the noblest creature and the focus). Hence, it is clear that both the imagination of the universe and the importance of the world centralization have been in the micro level represented in the painting under discussion and identifying this point reveals one of the other most important features of Persian Painting.
4. Discussion

4-1 Persian Painting

Every work of art with its tangible form is the development of that absolute truth which was revealed in the world. By creating his artwork, the artist realizes the truth of the form as he himself wishes. Copleston, explaining Plato's theory about art, says, "One of the characteristics of art, i.e. imitation, in the opinion of Plato, basically, implies composing the exact copy of a picture. Despite this fact, his words about 'true' imitation indicate that he mostly thinks in this way: First, a natural object is not a copy of an "example" picture because the example has its own order and the real picture which is understandable has its own order; as a result we can conclude that it is not a need for a work of art to be a copy of a real object through allegory analogy. In visual arts, the tangible form receives its structure and organization from the artist who is merged in the beauty of God and that’s why the result of immediate appearance is the real beauty. Therefore, form has two aspects in art; appearance and conscience. The conscience is the same real beauty and the appearance is the emergence and the result of the conscience. The more the form follows other than the real beauty in its organization, the more it will be far from genuine art. Also, the more the received real beauty by the artist is clear and intense, the more the form will be pure and high and in other words the work will be more artistic" (Copleston, 1996). Persian Painting which is rather known as "miniature" mostly has religious, romantic, and theosophical themes and is mainly linked with literature and the existing concepts in literary prose and poems; it is portrayed with the help of the artist's creativity, cleverness, and accuracy. Persian Painters were usually doing the job in royal workrooms in courts and under the supervision of the king.

Timurid period is the most important era of Persian Painting. Timurid Kings were art lovers and tended to have artists to be trained. Most of them changed their courts to a circle of artists, poets, and literati. Methods of Persian Painting which was founded in Shirazi School came to its highest point in Timurid period in Herat. Outstanding characteristics of Persian Painting like its strong link with Persian literature and theosophy caused the Herat Miniature School to be introduced as a model in the history of Iranian-Islamic Painting.

According to Miniature School of Herat, "what is fresh and new is the spirit of its components, especially of human bodies which have mostly lost their idle identity as dolls and seem to be struggling" (Grabar, 2005). Maulana Khalil and Ghiyasuddin were two of the representatives of Timurid School and their heritage was received by Kamal al-Din Bihzad (1545-1450) via artists like Master Mansour and Ruhollah Mirak (Pakbaz, 1999). Regarding Persian Painting with a history of thousands years, only two people are known; Painter Mani who lived in Sassanid Period (216-274 AD) and Kamal al-Din Bihzad a painter of Timurid and Safavid Periods (Bahari, 1996).

Themes like praising nature in an artistic way, describing the exact psychological states, and expressing theosophical passion came to their highest point thanks to his charming works. Surely, his humanitarian character formed due to his presence in cultural environment created by Amir Alishir Nava’ie and Abdurrahman Jami on one hand and being supervised by Maulana Mirak on the other hand. At the same time, Bihzad could innovatively compose some works beyond the traditional limits of painting through his deep insight and theosophical ideology. What made him a perfect artist was his thorough understanding when establishing a two-way interaction between the fancy atmosphere of Persian Painting and the world realities. In the world of painting, he sets the man, as the main pillar of any work, free from heavy dependence on literary texts and gives further expression to his role. The manuscript of the Quinary by Nizami, the manuscript of Layla and Majnun by Nizami, the manuscript of Zafarnameh, Bustan by Sadi, and some images of Sultan Hossein Bayqara, Turkish Prince, Dervish and Thinker are of his masterpieces; each one of them show the Painter's genius.
4-2 Geometry

Firstly, it is needed to have a look at the meaning of the word and then it will be elaborated and explained. In the first exposure with the meaning of this word in one of the most reliable mathematic and Islamic geometry references, i.e. Understanding Astrology by Al-Biruni, we found it defined as follows.

According to "Definite Proof", the Arabic equivalent of the word "geometry" means the size and shape (Tabrizi, 2012). And it is of principles of the science of mathematics which discusses measures and values (Thanvi) City, Year, & Publication. It is distinguishing numbers and their difference and the characteristics of the forms and shapes of the objects; it makes the detailed science of numerology become general and the science of the form which is approximate become real (Biruni Kharazmi, 1973). This definition has been explained as follows: The technique of arithmetic and astrology becomes logical and argumentative via geometrics and the characteristic of logic is that it changes the detailed themes and issues into general ones and makes the approximate ones certain and research-based (Biruni Kharazmi, 1973).

Other explanations about this word suggest its importance in ancient cultures like Greece and Egypt. The word "geometry" is the English equivalent for the Greek word "γεωμετρία" which means measuring earth (Hejazi, 2008). Geometry is the order of positions through measuring the relations among shapes. Geometry, arithmetic, and astronomy which are the sciences of the order of time were the main fields of education in ancient times. They aimed to teach geometry and training the mind was considered as a path through which the earth could understand the spiritual world, i.e. the spatial life of the skies. The usage of geometry was developing a method which put the universe in order and maintained this order. In ancient Egypt, flooding of the Nile inundated beaches and surrounding lands each year and destroyed the orderly packed lands. Egyptians believed that the yearly flooding was a sign of the periodic return of the water's primary fluctuation. When it was gone, they started to form and draw the borders again. It was called geometry and was seen as the principle of reordering the land again (Lawlor, 1989).

4-3 The importance of the geometric principles in the structure of Persian Painting Composition

By identifying and thinking about Persian Painting atmosphere, its elements, signs, and their relationship with Islamic Culture of Iran, we can see how the atmosphere of Persian Paining are in accordance with Islamic culture and philosophical system of Iran. Islamic Art, in each of its fields, could represent scientific and intellectual improvements of Islamic culture and one of its major characteristics is its connection with mathematics and geometry; the artist's tendency toward this category is evident everywhere. The structural analysis of Persian Painting is not that far from its philosophical concept. Today, modern analysts are aware of the fact that form and content are integrated. Abstractionism is one of the aspects of Persian Painting which can be mentioned as an explanation for this kind of painting as it is not a representation, mere naturalism, empiricism, and a personal view. In this regard, Hossein Nasr says, "Although unity which is the central idea of Islam is a concrete concept in itself, it is considered abstract by human beings. Similarly, mathematics is seen as an abstract concept compared to the world of tangibles. But, when viewed from the perspective of the reasonable world and Plato's world, guidance toward objective concepts which are objective facts is fixed. Just as the fact that all shapes are formed out of points and all figures arise from one. Similarly, every multiplicity is caused by the creator of the world who is single and unique. When shapes and figures are considered as what Pythagoras believes, i.e. as the natural aspects of unity, not mere quantities, they become a means to indicate unity out of multiplicity (Nasr, 2005).

What is introduced by Nasr is a theosophical and philosophical definition of Islamic Art which is not our concern in this paper. But, as will be mentioned in the following sections, when analyzing structurally, the main theme is considering the atmosphere of Persian Art to be geometric and mathematical and surely, Islamic culture has made essential steps in the development of mathematics and geometry. Without tending to review the broad themes and history of geometry, we want to mention some points which approve the significance of the application of mathematics and geometry in Islamic culture and civilization.
In Iran, geometry came into consideration before the appearance of Islam by ancient civilization of Ilam. Ziggurat Buildings (high temples) show the extent of progress and application of geometry. In Sassanid Period, too, the improvement of geometry can be seen in civilization. In Islamic Period, Sons of Musa ibn Shakir, i.e. Mohammad, Ahmad, and Hossein (9th century) were the first who investigate geometry by numerical calculations. Certainly, Abu Abdullah Muhammad ibn Khwarizmi who was engaged in scientific activities in late 8th and early 9th centuries was the best mathematician of his era. His two books in the field of geometry and mathematics are "Algebra" and "arithmetic". Other Islamic scientists like Abu al-Wafa' Buzjani, Abu al-Rayhan Al-Biruni, Brethren of Purity, and Omar Khayyam, too, has composed some works in the field (Bemanian, Okhovvat, & Bagha'ie, 2011).

Today, geometry is seen as a science which takes shapes and angles, their measures, characteristics, and relations into account. By making use of geometry, i.e. by observing scales and measures, artists seek to create harmonic and beautiful compositions.

It should be reminded that it is necessary for the artist who produces art in the framework of Islam to know geometry because Islamic Art is based on mathematics and geometry and in other words "in Islamic culture, artist and mathematician are actually the same" (Bolkhari Ghehi, 2009). The structure of Islamic Painting and Art in the smallest decorations which are displayed in Islamic imprints is to show that final unified composition of an artwork is based on geometric concepts; whether we look for it in a piece of arabesque and see it as an abstract expression of spatial geometric concept, or we come to a unified form out of multiplicity of these imprints and forms which itself can be one part of a larger unified one. In this way, as Keith Critchlow mentions, "the science of mathematics, especially geometry integrated with philosophical and inspiring themes and spiritual hints was appeared as a holy language which became the footstone of Islamic art as an image of the order which governs the universe (Critchlow, 2011).

The artist makes aesthetic choices among spatial elements when composing a painting. In fact, composition is one of the most important issues which the artist possesses in order to communicate to the audience. In the composition, two dimensions which exist on the surface of the painting are proposed as important and spatial displays, too, are arranged in terms of two-dimensional aesthetic scales. In this way, two-dimensional space of the surface and how to deal with it is the fundamental issue in the process of creating a work of art which is thought as the major procedure of structural and formalistic analysis. In the book "Composition in Painting", David Friend says, "Artists organize their concepts of space through thinking, not watching. We have accepted that they painted well and even better than us by using the limited and simple tools they had. Indeed, the great art of all the periods has survived because they not only show the spirit of the life of their time but also express the universal values. There are lots of ways to really create something. However, if two-dimensional harmonic relations are not established first in both painting and sketching, it will not be a work of art even if it suggests a three-dimensional image" (Friend, 2006).

Besides, the existence of a system or composition and the links among the components of the image are important in creating a work of art to the extent that the greatest art works have the greatest arrangements. However, sometimes, the composition is not detected by general audience as much as other things (Canaday, 2005). Different special compositions have emerged in Persian Painting which differentiates it from other types of paintings. After observing these works, it was concluded that the painter has tried to carefully arrange visual elements at two-dimensional level rather than focusing on a realistic representation. In Persian Paintings, the elements which are in the scene including human beings, nature, and objects, all have a mathematical identity and can be calculated by geometric shapes; these links fosters the unity of the work. The most significant geometric shapes which are included in the composition of Persian Painting are spirals, squares, rectangles, and triangles; they are hidden and visible, either completely or partially.
4-3-1 Spirals

One of the shapes introduced in hidden geometry is a spiral called logarithmic spiral or golden spiral. In some mathematical treatises, it is called Bernoulli spiral because the mathematician Bernoulli discovered that when the line segment which links the center of the spiral to one of its points, called radius vector, rotates around a central point in some directions, it increases in a geometric manner, while the angle that it makes with the direction of the movement (whatever it is), increases in an arithmetic (numerical) manner (Ayatullahi, 2006). The spiral is the symbol of origin, evolution, ascension, salvation, and shows going beyond oneself and joining the Eternal Power, God. A spiral is moving toward its origin; the one who is totally unique. Also, it indicates passing of time. Those shapes which evolve logarithmically always have the element of the passed time in them. As a result, they rather symbolize evolution of reason (Lawlor, 1989). Logarithmic spirals are called miraculous spiral by traditional geometricians due to their richness of geometric and algebraic coordination (Lawlor, 1989). The main structure of the composition of visual elements which is presented in Persian Paintings is using circular and curved compositions especially spirals; using such forms is in complete unity with their content and concepts. In compositions, spirals and rotary forms call the eyes to follow the elements along imaginary axes from center toward outer surface and vice versa.

4-3-2 Squares and rectangles

Usually, in paintings which contain horizontal inscriptions on the top or bottom of the frame or vertical ones on the right or left, harmony depends on the arrangement of these rectangles to some extent. Squares with their right angles and equal sides indicate a sense of being static, strong, solid, motionless and calm. Squares are mainly the shapes of place, and circles and especially spirals are mainly the shapes of time. Reality can be expressed through squares. Circles and squares (spheres and cubes) always indicate sky and earth and the link between them connotes their connection (Hohenegger, 1987). Squares, the easiest forms of creation to be embodied, are the representatives of quantity and in the limitations of the earth while circles are the representatives of skies and possess their dignity. Squares and circles are integrated via triangles which represent a mixture of these characteristics. Golden rectangle is the one whose sides have the golden ratio. If we draw such a rectangle and separate a square with sides of width of the rectangle, we will have a golden rectangle again. We see the same if we repeat the same times and times (Al-Buzjani, 1990-1997).

Figure 1: sketching a spiral and a golden rectangle by using irrational numbers (Al-Buzjani, 1990-1997)
Numbers like $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ ... were called irrational numbers by the Pythagoreans; they can be drawn but cannot be expressed. Rectangles with the ratio $\frac{a}{b}$ which equals $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{3}}$, $\frac{1}{\sqrt{5}}$ ... that is $\frac{1}{\sqrt{n}}$ was called dynamic rectangles by Hambidge. In the rectangle $\sqrt{2}$ which is shaped with the help of generative index square, if the shorter side, i.e. its main side which is one side of the index square, equals one, the longer side equals the square's diagonal, i.e. $\sqrt{2}$; almost 1.4142. This rectangle has special characteristics. For example, the diagonal of the rectangle $\sqrt{3}$ equals $\sqrt{4}$=2 and the diagonal of the rectangle $\sqrt{4}$ equals $\sqrt{5}$ and so on. The way these rectangles are shaped and "ratios" which increase in each time of expansion are very useful. The valuable points of the rectangle $\sqrt{2}$ include the followings: where the rectangle's diagonal meets the section of the index square, where the rectangle's diagonal meets the diagonals of the half rectangles, where the diagonal of the index square of one side meets the section of the index square on the other side, etc.; These are called the composition's and rectangle's golden points (Ayatullahi, 2006).

Guidelines are the lines which guide the movement and the way of the elements' dispersal or centralization which compose the work; the basic geometry of the work comprises the collection of these lines (figure 4-10). Appropriate use of these lines helps the artist to convey his ideas as he should and develop the work in a way that attracts the audience and makes them search in the image. Guidelines set the right place of the elements which make an image including points, lines, and spaces to bring about a full effect. These guidelines are the followings:

1. The sections of the index square in the whole rectangle
2. The diagonals and the cross-cuts of the index square
3. The diagonals of the whole rectangle
4. The diagonals of the rectangle which is the supplement of the index square
5. The lines which makes an angle of 90° with the diagonals of the whole rectangle, from each of their endpoints.
6. The sides and the diagonals of the golden pentagon surrounded in the index square (extending them to meet the sides) which are called force lines, too.
7. The lines which links other points with the points where the extended sides and the diagonals of the golden pentagon meet the diagonals of the whole rectangle.
8. Line segments which are drawn parallel to the sides or diagonals; each with one specific endpoint, i.e. the inside golden points of the whole rectangle.
9. Free lines (diagonal ones) which link the golden point with other points.
10. Vertical and horizontal lines which are drawn in the gathering centers of the lines and parallel to the sides (Ayatullahi, 2006).

One of the fundamental topics of geometry is the category of scales which not only scientifically but also generally is evident in the whole world and is very important. All shapes, surfaces, and lines are arranged according to the nature and reflect beautiful ideal systems. Order and proportionality to space are like rhythm to time and harmony to voice (Ardalan & Bakhtiar, 2000). Al-Biruni defines proportionality in this way: it is when two or more ratios come in harmony with each other; at least three values are needed. The first sample is the ratio of five to one. So, the first is one fifth of the second and the second is one fifth of the third. So two ratios are achieved among three values (Biruni Kharazmi, 1973). The golden ratio (Golden proportion, Divine proportion, and divine section) is shown with the symbol $\phi$. Phi is the 21st letter of the Greek alphabet. It is equal to the fraction $\frac{1+\sqrt{5}}{2}$; almost equal to 1.618. It is called golden number or magic number, too.

![Diagram 1: Golden Ratio (Reference: the researcher)](image)
4-4 Hidden Geometric System

Geometry is expressed in the form of a symbolic language which reveals the hidden geometric composition of the universe and the relations among its elements. It is a path beyond the images to what cannot be seen; it is understanding the way in which the first and the last get to each other, how the components become a whole, and vice versa. Knowledgeable and clever painter conveys the meaning based on a rule-governed structure which surely has a core. The audience receives the meaning; but, it is just the curious and well-informed audience who decodes the procedure followed by the artist in order to get to the core which determines the relations among the components and the artist's plan. In this case, by observing the relations among the components and exploring their mathematical relations, understanding the visual data occurs in its best way. Now, the most important task is organizing the observed visual information. The most reliable way to do this is investigating geometrical structures and drawing them carefully by using obtained measures of the examined sample. Perhaps the same has happened to the artist during art creation, i.e. he has involved in a complicated mental process, rejecting and accepting the same options. He has developed the best links in his miniature's infrastructure thanks to his artistic talents and mathematical knowledge. In the same phase, i.e. mental visualization, the artist or painter is able to examine different arrangements of visual elements freely and in different ways through trial and error; an appropriate arrangement which conveys the meaning well. Maybe, at first, details are not considered and the main relations in the link are formed with the main characters of the painting. However, each of the visual elements finds its role in the composition in the next phases.

Primary and major elements which make the composition creation possible to start are the same points, lines, surfaces, shapes, etc. In other words, the simplest elements play the most fundamental roles. That’s why, from the beginning, a space is needed for these geometric events to happen in its format. So, drawing a vast network was the first to be done; a network of horizontal and vertical axes, whose generator, in the first place, is recognizable main borders and then smaller borders which are not visible directly but emerge out of the main borders or the divisions of the main spaces into smaller pieces. These visual units have some guidelines and links which show the logical layout of the elements clearly in the early steps of analysis. After drawing these units through mathematical rules, a large number of further subordinate axes are obtained in each of the main borders which are very helpful. In this crowded set of lines and shapes, that some of the axes are close to each other and the points are mixed makes the identification of geometrical relations difficult and time-consuming. That’s why it was tried to keep the presence of all main axes fixed and depending on the degree of importance of subordinate ones, we omitted some of them for the final show of the geometric structure (figure 3). These lines can be traced with different types and colors in the images. In these primary steps, thinking a bit about the obtained networks, we can find pleasing evidences. For example, vertical, horizontal, diagonal axes and their commissure where they meet determine the beginning or end of many elements of the images such as main figures or architectural components (arches, inscriptions, etc.). Now the work speaks its content; in fact, the purpose of exploring the formal nature of these works is hearing this sweet melody. Besides, there are a lot more of this. Finding close relations among the elements of the miniature is a kind of entering a crucial phase that has been one of the most preliminary steps taken by the artist. However, mainly paying attention to the other aspects like charming colors and designs (not the lines and the relations among them) have been very important. It’s interesting to know that the remained treatises of these periods includes explanations and descriptions more on technical aspects such as the way of making paper, color, or paintbrush but less on such things. If there is something about geometry, it is regarding the consideration of sizes when drawing charts and geometry of the patterns in decorations; almost nothing is found concerning the way of designing patterns and geometric structures of miniatures. As a result, it remained a valuable well-kept secret in the minds and hands of unique Persian Painters.

Establishing the structure of a miniature has some rules which can be set as a guide to the core of the structure. Usually, the main point of the structure, which is seen as the focus and the central point in some cases, is the most important zone of the composition. It means that the closer we get to the core of the image, the more rule-governed becomes the relations among components and the more their geometric accuracy is fostered. Hence, the geometric infrastructure’s core can be looked for in the same area of focus.

To avoid entering the world of numerous rules of geometry, we investigate these concepts in the form of the most fundamental shapes. Therefore, the main structures including spirals/rotary movement (figure 5), squares (figure 6), and rectangles (figure 7) were selected in order to scrutinize the miniature schematization after identifying them in an elaborate system of lines which has been drawn very carefully.
To start, each of the primary visual forms and elements, as the basis to establish the process of the analysis of the geometric structure, were investigated. The mentioned main forms and their different types were set as the scale of geometric evaluation because all of these basic shapes are the source for extracting visual structures in different kinds of arts like Painting, architecture, and sculpture and whatever form is expressed via them. Each of them has its own special characteristics. All of them can be described easily and their primary sketch can be drawn simply. However, here, we're talking about sketching them geometrically, i.e. by applying mathematical rules. Because when a geometric structure appears in a miniature based on a spiral form, it is not seen as a simple spiral arc; rather, what is seen is a collection of vertical, horizontal, and diagonal lines, proportional angles and specific values and what solves the puzzle of the geometric infrastructure for us is the arrangement of visual elements which is in accordance with the interior space of the form or its angles and guidelines. If we line these geometric standards up, they are put in the following turns:

Regarding the geometric structure based on rotary movement, among different kinds of spirals including (symmetrical spiral or Archimedean spiral, hyperbolic (reciprocal spiral) or logarithmic (equiangular) spiral or Fermat’s (parabolic) spiral, lituus spiral or Golden spiral, logarithmic spiral (equiangular) was pursued. Regarding the geometric structure based on squares, a square named "index square" or "generator square" which has the ability of making rectangles based on the golden ratio with different proportions of (1, 1, 1, \frac{1}{1.618}, \frac{1}{500}, \frac{1}{1.414}, \frac{1}{1.516}) was used. Geometric structure based on rectangles was examined by using the rectangle with the golden ratio on the basis of irrational numbers (\sqrt{2}, \sqrt{4} ...). Each sample was pursued and sketched, geometrically, by considering the proportions of each in order to make the harmony of geometric structures and combinations clear. Then, the precise degree of all angles, lengths and widths, the main and subordinate visual units of lengths and widths (of squares, rectangles, circles, and other shapes) was measured. As each of them had relations based on the golden proportions, it was a ray of hope; it announced the existence of a geometric infrastructure. After that, each of the multiple structures whose existence was informed by the relations was drawn on the geometric network of the infrastructure of the image in the form of a modular and the proportions were identified. It was here that the hidden order of each one of the components freed the whole work from disorder and the arrangement, which certainly means putting everything in its own place, caused a good balance and the ultimate perfection of the work was achieved based on rule-governed principles.

Sometimes, in the connection shared by the content and the form of the work, a concept such as a dominant force determines the relationships and the distances which were trapped in the limitations of formal plays. It doubles when art creation is done because someone has ordered it; when the artist works on a painting, if not as an order, is not to think whether the one who ordered the work likes it or not. So, he concentrates on his own pleasure and satisfaction and works on his skills; then the work will be quite personal. But, when someone orders a work (it can be a person of high rank like a king), the artist, mostly, does not make choices based on his own taste and perhaps he follows up-to-date sciences and the public taste. That’s why each work can be viewed as an example of the attitudes and beliefs of the people of a period, era, or social class. In Timurid Period, i.e. one of the historical periods of art in Iran, the existence and the role of a king (Imam, Sheikh, Perfect Master) was really important as the focus of attention, the organizer of all the affairs, and the guide of culture and art; also, some of them were art lovers and supported the artists. It seems logical if such a person expects the team of the court painters at the library, royal workroom, to compose great works and organize the elements of their miniature carefully and properly.

The phrase "following the creation system of the world" means and leads to the natural harmony of all of the particles and the symbiotic relations of the elements at the macro level. Here, too, the king, as a perfect man, is considered the main hero in the image and people thinks that he has the highest rank. The world has been created in accordance with the creation rule and the man is the noblest creature. So, the king, as the source of absolute truth and at the highest position, comes to the focus of attention to be the criterion for sketching a miniature "a celebration" and measuring it. As it is seen in the last four pictures, all the components of the main single figure which is the image of Amir Teimur has harmonious proportions with each other (from the feather on the crown to the tip of the shoe); these proportions have been kept in all the dimensions and vertical, horizontal, and diagonal measures (figure 8). Not only the major components of the image but also the smaller ones like the face, the crown, the parts of the clothes, etc. have been drawn based on the coordination principle. Even it seems that less important elements such as the visible part of the rod (the part which is not covered by the turban) of the Ghelzelbash hat has a systematic proportion with the part which is covered with the turban or the upper part of the shoe, from the leg to the sole of his foot and so on (figure 9).
There are other similar examples but the most interesting is that the measures of the image of the king develop a completely coordinate and connected system in the whole composition in which each significant measure is in accordance with at least four or five related measures (figures 10 and 11).

In this paper, regarding the selected miniature, a piece of information was given in brief and geometric analysis was presented to support what was mentioned. The case study of the research entitled "a celebration after Timur’s return to Samarqand" is one of the valuable Persian Artworks by Kamal al-Din Bihzad which belongs to Timurid Period and Herat School and was painted in 1531. This paining with the dimensions 27*23 centimeters depicts a scene of a royal feast of Zafarnameh Timurid and is preserved in Tehran Museum of Contemporary Art. Timurid Zafarnameh is a book about the life, expeditions, and the historical description of the conquests of Teimur and one of the most important historical copies which was written in 15th century (Morrison, 2001). Zafarnameh or Teimurnameh, the Persian version, has the dimensions of 23*37 and 750 pages and 24 miniatures of feasts and wars and some scenes of court ceremonies (Semsar, 2000). Pakbaz writes about the artists who were involved in creating this copy in this way, "in the last page of the illustrated copy of Zafarnameh, Sultan Mohammad Noor is introduced as the calligrapher, Mir Azad as the designer of the layout religion, and Bihzad as the painter (Pakbaz, 2011)."
The analysis of the geometric structure of the miniature

Figure 3: the basic geometric infrastructure (main and subordinate borders, main and subordinate vertical and horizontal axes)
Figure 4: the coordination of the main image and the basic geometric infrastructure

Figure 5: logarithmic spiral with its guidelines
Figure 6: the geometric structure based on the index square, or the generative square, and the rectangle derived from it

Figure 7: the geometric structure based on the golden rectangle
Figure 8: repetition of the scales of lengths and widths of the standard image in the whole composition

Figure 9: the existing proportions of the standard image
Figure 10: the coordination between the scales of the standard image and the existing scales of the main and subordinate axes of the composition

Figure 11: comparing the king's image with the whole miniature
Table 1: the extent of the use of geometric principles and their coordination in the miniature "a celebration after Timur’s return to Samarqand"

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<thead>
<tr>
<th>The basis of the hidden geometric system</th>
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<tbody>
<tr>
<td>1 Logarithmic spiral</td>
<td>4 Expanding the lengths and widths of the image of the king</td>
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<tr>
<td>Locating the main elements and figures of the miniature in coordination with spiral rotation axes and their distances</td>
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<tr>
<td>2 Golden ratio in the rectangles resulted from the index square based on the golden number</td>
<td>5 The coordination of the main and subordinate borders of the composition with the hidden proportions of the image</td>
</tr>
<tr>
<td>$\frac{1}{1.360}$</td>
<td>$\frac{1}{1.414}$</td>
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<td>$\frac{1}{1}$</td>
<td>$\frac{1}{1}$</td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>+2</td>
<td>-</td>
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<tr>
<td>3 The golden ratios in the shaped rectangles based on irrational numbers</td>
<td>6 Comparing the standard image with the miniature</td>
</tr>
<tr>
<td>$\sqrt{2}$</td>
<td>$\sqrt{3}$</td>
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<td>-</td>
<td>+3</td>
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(Reference: the researcher)

The explanation of the numbers and signs mentioned in the table:
Using and not using the mentioned geometric basis has been shown with the signs – and +. To state the times of repetition of one single unit, the figures 1, 2, 3… were used. It is needed to mention that in section 4, the number of repetitions of the vertical and horizontal measures is presented in accordance with the scales of the measures of the king's image and in section 5 which is related to the coordination of the main and subordinate borders of the composition with the hidden scales of the king's image, the extent of usage was presented with the help of numbers; the number on the right shows the number of the main coordinated axes and that on the left is the number of the subordinate axes of the main borders which are in accordance with the scales of the standard image.
References


